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Gluino signals in 4jet events and vertex tagging at LEP I

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Abstract. Heavy flavour tagging provides a broad range of possibilities in testing QCD features at LEP. We present here a study of 4jets events at LEP I where the so-called light gluinos could be directly produced. We show that microvertex techniques offer a unique chance to exploit simple kinematical distributions in order to optimise the signal coming from gluino production with respect to the background of ordinary QCD events. Our results indicate that experimental analyses along the lines suggested here can exclude or reveal the presence of a gluino for masses up to 10 GeV and lifetimes below 10^{-9} sec. We also point out that a large fraction of gluino events could decay in configurations carrying large missing energy, so to escape the usual selection criteria of 4jet samples. In our study, mass effects of quarks and gluinos have been taken into account exactly. Our results are independent from both the jet algorithm and its resolution parameter.

1 Introduction

Supersymmetry (SUSY) predicts the existence of a spin 1/2 partner of the gluon g, the so-called gluino $\tilde{g}$. Like the QCD gauge boson, it is neutral, it is its own anti-particle (i.e., it is a Majorana fermion) and its coupling to ordinary matter is precisely determined in terms of the usual QCD colour matrices and the strong coupling constant α_s [1]. However, in contrast to the gluon, whose mass is predicted to be zero by the theory, that of the gluino $m_{\tilde{g}}$ is a priori an arbitrary parameter, and so is its lifetime $\tau_{\tilde{g}}$.

Many searches have been carried out in order to detect or rule out such a particle. A detailed survey, including the description of various experiments, can be found in [2]. In particular, light gluinos should be directly produced in 4jet events at LEP I.

Motivated by the advances in 4jet analyses based on heavy flavour identification, we further elaborate the study of [3] to cater for a wider range of light gluino masses and lifetimes. The use of μ-vertex devices [4, 5] provides an independent procedure to settle the ongoing controversy around the light gluino scenario, if one considers that long-lived gluinos might produce 4jet events with detectable secondary vertices [6, 7]. We assume them to be long-lived enough to be tagged by present experimental vertex tagging methods. With simple invariant mass cuts based on the different kinematics of the partons in the final state, one can obtain a signal identifiable as a clear excess in the total number of vertex tagged 4jet events, in percentage well beyond the uncertainties related to non-perturbative as well as to higher order perturbative effects. We also consider the possibility of 'loopholes' in recent analysis carried out by the ALEPH collaboration [8] and by de Gouvêa and Murayama [9], which could undermine the validity of the results obtained there. We are especially concerned with the fact that gluinos might decay mainly into missing energy, so that the $q\bar{q}\tilde{g}\tilde{g}$ events are not recognised as 4jet events. As a matter of fact, the ALEPH analysis explicitly assumes that this is not the case whereas [9] only considers the case in which the gluino does not decay inside the LEP detectors.

The plan of the paper is as follows. In the next section we review the status of the light gluino window and put our work into context. Next we describe our calculations, and in Sect. 4 we devote some space to discuss possible tagging procedures of SUSY events. In Sect. 5 we present our results and Sect. 6 contains some brief conclusions.

2 The light gluino window

Our present knowledge about gluinos is summarised in Fig. 1, which shows the excluded regions in mass and lifetime of the SUSY fermion as they stood in 1993-1994[1]. At that time, it was clear that relatively long-lived and light gluinos (decaying into a 'photino', or more correctly, into the lightest neutralino) were not yet excluded by the experiment.

[1] We would like to thank the authors of [10] for their kind permission of exploiting here one of the figures of their paper

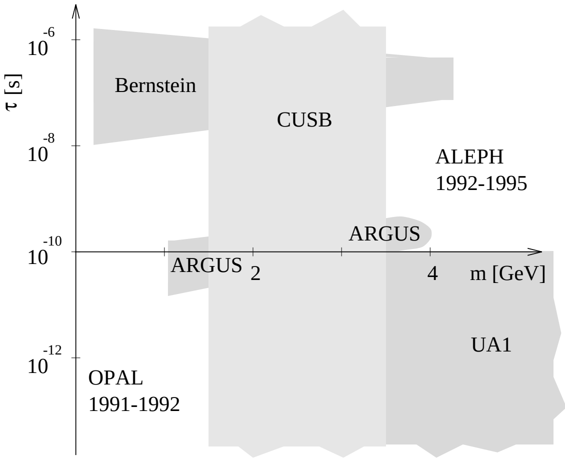

Fig. 1. Excluded regions of gluino masses and lifetimes (*shaded areas*), from [10]

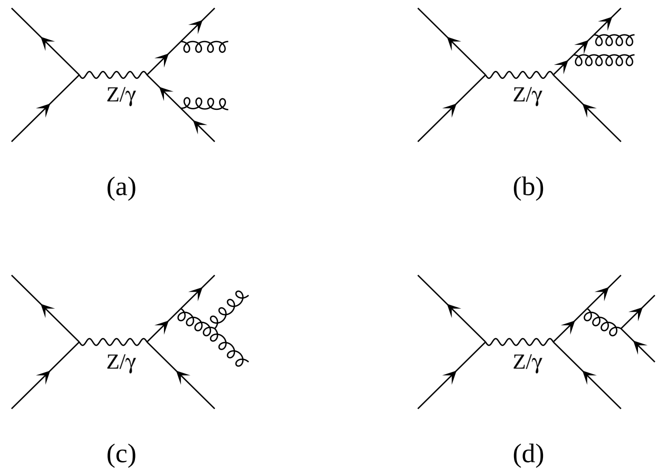

Fig. 2. Feynman diagrams at lowest order contributing to 4jet production in e^+e^- annihilations in ordinary QCD: $2q2g$ contribution (a,b and c); $4q$ contribution (d). All possible permutations are not shown

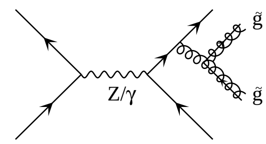

Fig. 3. Additional Feynman diagrams at lowest order contributing to 4jet production in e^+e^- annihilations in QCD+SUSY: $2q2\tilde{g}$ contribution. The other permutation is not shown

In the theory it is natural for gluinos to be much lighter than squarks if their mass is induced radiatively [11]. Furthermore, gluino and lightest neutralino masses are naturally less than few GeV if dimension-3 SUSY-breaking operators are absent from the low energy theory [12], although the compatibility with a light gluino window in Supergravity models is strongly dependent on the dynamics of the breaking mechanism of the electroweak symmetry [13,14]. Since these values of $m_{\tilde{g}}$ and $\tau_{\tilde{g}}$ were within the reach of already operating accelerators [15], the regions identified by the white areas in Fig. 1 started receiving some attention in those years [16]. In particular, it was noted that if the gluino is so light, it should be directly produced at LEP I: either in 2jet [17] or in 4jet events [18].

An immediate interest in this possibility raised. This was also motivated by the 'historical' discrepancy between the value of α_s determined by low energy deep-inelastic lepton-nucleon scattering and that measured by the e^+e^- CERN experiments [19]. In this respect, although the discrepancy between the two values of α_s was statistically small [20] and also has slightly decreased in the latest measurements [21], it was speculated that the evolution of the strong coupling can be slowed down by a contribution to the β function of a new, coloured, neutral fermion: indeed, a light gluino.

The search for SUSY-signals intensified then at LEP I. Studies in other contexts, such as the influence in the Altarelli-Parisi evolution of the structure functions [22] or the so-called '3+1' jet events at HERA [23], were also pursued. However, the effects are there too small to be tested using the present experimental data. More recently, extensive searches for light gluino signals have been carried out at the Tevatron [24]. As for LEP I, the strategy adopted was to search for light gluinos in the context of the so-called QCD colour factor analyses in 4jet samples [25]. The basic idea is to measure the fundamental colour factors of QCD, that is, C_A, C_F (the Casimir operators of the fundamental and adjoint representations of the gauge group $SU(N_C)$) and T_F (the normalisation of the generators of the fundamental representation). In QCD (i.e., $N_C = 3$), one gets $C_A = 3$ and $C_F = 4/3$. The factors C_A, C_F and T_F represent the relative strength of the couplings of the processes $q \to qg$, $g \to gg$ and $g \to q\bar{q}$, respectively (see, e.g., [26]). The analytical formulae of the cross section $\sigma(e^+e^- \to 4\text{jet})$ for massless particles were computed long ago in [27]. The strategy is to compare the theoretical predictions to the data, by leaving the colour factors as free parameters to be determined by the fit. In practice, one of these, e.g., C_F, is absorbed in the normalisation of the cross sections leaving two independent ratios C_A/C_F and T_R/C_F, with $T_R = N_F T_F$, being N_F the number of active flavours.

The experimental analyses are preferentially based on angular correlations between jets [28], as they are sensitive to differences between the $2q2g$ (Fig. 2a–c) and the $4q$ (Fig. 2d) component of 4jet events. Light gluinos would enter in 4jet events via diagrams of the type depicted in Fig. 3, in the process $e^+e^- \to q\bar{q}\tilde{g}\tilde{g}$ (through a $g^* \to \tilde{g}\tilde{g}$ splitting). Note that gluino production in 4jet events via squarks splitting into quark-gluino pairs is very suppressed due to the large value of the lower limits on the squark masses, so is also the case in 2jet production through squark loops [17]. As gluinos are coloured fermions, their contribution would enhance the part of the 4jet cross section with angular structure similar to that of $4q$ events. Naively then, one could say that the total number of flavours N_F of the theory is apparently increased, such that, a SUSY-signal can be revealed in the form of an enhancement of T_R, with respect to the predictions of pure QCD. The results of those analyses were that, although the experimental measurements were in good agreement with QCD, it is was not possible to exclude the existence of a light $\tilde{g}$ (see, e.g., [29]). In particular, gluinos with a mass of at least 2 GeV yield an expectation value for T_F/C_F that was within one standard deviation of the measured one. Even the extreme case of a massless gluino (for which $T_F/C_F \approx 0.6$) would have brought the predictions only slightly beyond the upper experimental region of 68% confidence level (CL) given in [29]. Therefore, after those studies, the experimental constraints on the gluino mass and lifetime could still be summarised by the plot in Fig. 1.

The reason why the LEP analyses showed a limitation in putting stringent bounds on the existence of light gluinos was that contributions to the total cross section of SUSY events are small and further reduced with respect to the ordinary QCD rates when mass suppression is taken into account. In particular, gluino effects on the total number of 4jet events were comparable in percentage to the systematic uncertainties related to jet hadronisation process and the uncalculated next-to-leading order (NLO) corrections through the order $\mathcal{O}(\alpha_s^3)$.

To overcome these systematic limitations, it was recently proposed in [9] to consider the colour factor C_A/C_F sufficiently well known as to be taken for its QCD value, 9/4. The other factor is also partially constrained, by the fact that there are clearly five quark flavours which are active in the di-jet Z decays. Therefore, if one wants to pin down possible gluino effects, one should allow variations of T_F/C_F only above the value 3/8. Armed with these two new constraints, [9] obtained an improved bound on $m_{\tilde{g}}$. In particular, it was shown that a gluino mass $\lesssim 1.5$ GeV is apparently excluded at more than 90% CL by the 1991-92 OPAL data [29]. A new experimental analysis carried out by ALEPH in [8] obtains results along the same lines. They give new measurements of the QCD colour factors using all the data collected from 1992 to 1995 and obtain excellent agreement with ordinary QCD along with a new 95% CL constraint $m_{\tilde{g}} > 6.3$ GeV on the gluino mass[2]. This stringent limit was achieved thanks to a dedicated treatment to reduce Monte Carlo (MC) uncertainties related to the hadronisation process of the partons and to the fact that meanwhile preliminary results of the NLO corrections to the 4jet rate had become available (the complete calculation has been presented very recently [31]). On the one hand, several different models of parton fragmentation were compared to each other with different parameter settings, on the other hand, it was clear that NLO results have a strong impact on the 4jet rate, but very small influence on the shape of the angular distributions used in [8].

Although these results represent a clear improvement, the analyses of [8,9] are still based on the traditional method [25] of ordering the jets in energy and identifying the two most energetic ones with those originated by the quarks produced in the Z decay. In fact, the angles which are generally used (see [28] for the exact definitions) in measuring the colour factors of QCD require in principle the identification of primary and secondary partons. In practice, the above assumption is often incorrect and the sensitivity of the experimental distributions to the QCD colour factors is considerably reduced. In this respect, it is worth recalling that, e.g., in $Z \to q\bar{q}gg$ events the percentage of events in which the two lowest energy partons are both gluons is only $\approx 53\%$ [32].

A possible improvement of the 'energy ordering' procedure was advocated in [33], where samples of 4jets with two jets tagged as heavy flavour jets (i.e., c- and especially b-quarks) [4] are considered. In this way, one gets a greater discrimination power between $q\bar{q}gg$ and $q\bar{q}q'\bar{q}'$ events, for two reasons. First, one is able to distinguish between (heavy) quark and gluon jets, thus assigning the momenta of the final states to the various particles in a more correct way, as heavy quarks are mainly produced as primary partons. Second, $q\bar{q}gg$ event rates are reduced by the heavy flavour selection by a factor of 3/5 with respect to the $q\bar{q}q'\bar{q}'$ ones. Therefore, the $4q$ signal is enhanced by almost a factor of two and the differences between the quark and the gluon components can be more easily studied.

The DELPHI collaboration is the only one to date (to our knowledge) that has resorted to flavour identification techniques to analyse 4jet events [34] (similar studies in the case of 3jet events are performed in [35]). They examined the data collected in the years 1991-1994 , from which a total of 11,000 4jet events with at least two heavy quark jets were selected. The typical efficiency was 12% with a purity of events where all jets are correctly assigned of about 70%. Note that neural networks were employed to combine the information on high transverse momentum leptons, large impact parameters and energy ordering of the jets (see, e.g., [4] for a review about techniques of heavy flavour identification). Their results have been presented recently [36]. The important outcome is that with the new selection strategy the errors are substantially reduced compared to previous analyses [25,29], especially for T_R/C_F. In general, the result was found to be in good agreement with the QCD expectations, but no new constraint on the mass of a possible light gluino was given at that time. Further analyses along the same lines are currently in progress [37]. As we shall see later on, our findings further support the relevance of such approaches.

3 Calculation

The Feynman diagrams describing at tree-level the reactions

$$e^+ + e^- \to q + \bar{q} + g + g, \quad (1)$$

$$e^+ + e^- \to q + \bar{q} + q' + \bar{q}', \quad (2)$$

$$e^+ + e^- \to q + \bar{q} + \tilde{g} + \tilde{g}, \quad (3)$$

are shown in Figs. 2–3. In the present analysis we have computed the matrix elements of processes (1)–(3) with

[2] Further indications towards the exclusion of somewhat lighter gluino masses come from studies of the running of α_s at higher orders [30]

the same FORTRAN generator used in [33,38], which takes exactly into account all masses and both the γ^* and Z intermediate contributions. Mass effects in 4jet events are important, as repeatedly recalled in the literature [33, 38–40], especially if heavy flavour selection is performed. For this study, the above program has been also checked against the one used in [39] in the appropriate limit (i.e., when the masses along the fermion lines attached to the γ^*, Z propagator are neglected). For the details of the numerical computation as well as the explicit helicity amplitude formulae, see [38].

We have analysed processes (1)–(3) adopting four different jet-resolution criteria for resolvable partons. We have done so in order to investigate the independence of our conclusions from the actual criteria employed and to check whether any of these shows better features for the analysis of light gluino contributions. The jet algorithms are identified through their clustering variable y_{ij}. They are ($\sqrt{s} = M_Z$): the JADE scheme (J) [41] based on the 'measure'

$$y^J_{ij} = \frac{2E_iE_j(1-\cos\theta_{ij})}{s}, \tag{4}$$

and its 'E' variation (E)

$$y^E_{ij} = \frac{(p_i+p_j)\cdot(p_i+p_j)}{s}, \tag{5}$$

the Durham scheme (D) [42]

$$y^D_{ij} = \frac{2\min(E_i^2,E_j^2)(1-\cos\theta_{ij})}{s} \tag{6}$$

and the Geneva algorithm (G) [43]

$$y^G_{ij} = \frac{8}{9}\frac{E_iE_j(1-\cos\theta_{ij})}{(E_i+E_j)^2}. \tag{7}$$

For all of them the two (pseudo)particles i and j (with energy E_i and E_j, respectively) for which y_{ij} is minimum are combined into a single (pseudo)particle k of momentum P_k given by the formula

$$P_k = P_i + P_j. \tag{8}$$

The procedure is iterated until all (pseudo)particle pairs satisfy $y_{ij} \geq y_{\rm cut}$. The various characteristics of these algorithms are summarised in [43]. In our lowest order calculation, the 4jet cross section for a given algorithm is simply equal to the four parton cross section with a cut $y_{ij} \geq y_{\rm cut}$ on all pairs of partons (i,j).

It is worth noticing that, in the recently calculated NLO corrections to the 4jet rates [31], the much forgotten Geneva scheme has been shown to be particularly sensitive to the number of light flavours as well as to exhibit a small scale dependence. In this respect, the G scheme may be more suitable than others in enlightening possible gluino contributions in the experimental sample.

Concerning the numerical part of our work, we have taken $\alpha_{em} = 1/128$ and $\sin^2\theta_W = 0.23$, while for the Z boson mass and width we have adopted the values $M_Z =$ 91.1 GeV and $\Gamma_Z = 2.5$ GeV, respectively. For the quarks we have: $m_c = 1.7$ GeV and $m_b = 5.0$ GeV while the flavours u, d and s have been considered massless. We have varied the gluino mass $m_{\tilde{g}}$ in the range between 0 and 20 GeV. Finally, the strong coupling constant has been set equal to 0.115[3].

4 Tagging procedure

In this section we describe possible signatures of long-lived gluinos in 4jet events at LEP I. We will resort to the fact that gluinos should produce displaced vertices and offer a complementary tool to the ALEPH study [8]. We also single out those combinations of SUSY parameters which could induce gluino decays into a large amount of missing energy. In this situation, the contribution of SUSY events to the 4jet rate as selected in the ALEPH analysis would be considerably reduced. Since we are implying that gluinos decay inside the LEP detectors, our considerations will only apply to the case of lifetimes less than 10^{-9} sec or so. In terms of mass, we will focus our attention on the two regions: (i) $0 \lesssim m_{\tilde{g}} \lesssim 1.5$ GeV; (ii) $m_{\tilde{g}} \gtrsim 3.5$ GeV.

Long-lived gluinos can be operationally defined as those which hadronise before decaying. An inevitable consequence is that they live confined into bound states, generically called R-hadrons [16]. The lightest of these would probably be the neutral, flavour singlet $(\tilde{g}g)$ and $(\tilde{g}uds)$ hadrons.

If the gluino mass falls in the range (i), decays into the minimum hadronic mass, i.e., $(\tilde{g}g) \to \tilde{\gamma}\pi\pi$ and/or $(\tilde{g}uds) \to \tilde{\gamma}\Lambda$, maximize the missing energy and, therefore, the SUSY signal in the ALEPH analysis can be well attenuated. If $\tau_{\tilde{g}} \gg \tau_b$, SUSY events should show displaced vertices at a distance d significantly larger than the decay length produced by a b-quark (i.e., 300 μm or so). If $\tau_{\tilde{g}} \approx \tau_b$ then the 'degeneracy' discussed in [3] would occur between heavy quarks and gluinos, so that the double vertex tagging procedure combined with appropriate kinematic cuts (see Sect. 4) should help to disentangle the gluino contribution. If $\tau_{\tilde{g}} \ll \tau_b$, then it may not be useful to look for detached vertices, since these would not be detectable for $(\tilde{g}uds) \to \tilde{\gamma}\Lambda$ and would be too close to the interaction region. In general, for a gluino with mass below 1.5 GeV, the fragmentation function should be similar to that for the charm quark, i.e., with $\langle z\rangle < 0.5$, or even softer if the mass is very light. Hence, there is a maximum missing energy possible, and it may be that this whole region is excluded by the ALEPH analysis. If not, there must be significant missing energy correlated with the directions of the soft visible gluino jets. A combination of a suitable missing energy distribution with the ALEPH analysis should be able to find or exclude such a light gluino.

In the second regime (ii), the gluino would presumably fragment into a $(\tilde{g}g)$ or $(\tilde{g}uds)$ hadron with a fragmen-

[3] Our results will not be affected by the actual value of α_s, as we will be interested in the end in relative differences between ordinary QCD and QCD+SUSY event rates

tation function perhaps similar to a b-hadron, i.e., with $\langle z\rangle \sim 0.75$. This hadron would then decay into a $\tilde{\gamma}$ plus hadrons with a distribution similar to that for $\tilde{g} \to \tilde{\gamma} q\bar{q}$. The missing energy would be maximized if the mass of the $\tilde{\gamma}$ is close to that of the $\tilde{g}$, but a limit to the mass difference is set by the requirements that the decay occurs inside the tracking volume and that the squark masses be reasonable. The charged multiplicity distribution should be similar to that for $e^+e^- \to q\bar{q}$ at the same $q\bar{q}$ mass, and events occurring inside $R \sim 0.5$ m with non-zero charged multiplicity should be observed with high probability. Such events would have two hard jets, two soft jets, missing energy, and two largely detached vertices with $d \gg 0.3$ mm in the directions of the soft jets. If the rate corresponding to these events is rather poor, then it is conceivable that the LEP collaborations might not have noticed them so far.

5 Results

5.1 Production rates

In this section we compare the production rates of SUSY events, as a function of the gluino mass in the range between 0 and 20 GeV, to the yield of ordinary QCD events. We do this with and without assuming vertex tagging (the titles $e^+e^- \to$ VVjj and $e^+e^- \to$ jjjj in the forthcoming plots will refer to these two cases, respectively). In the results of the cross sections for the untagged case a summation over all the quark flavours (massless and massive) is implicit, whereas for the tagged case we will consider the detached vertices as produced by gluinos and b-quarks only, thus neglecting the case of c-decays[4], and sum over the remaining quark flavours.

In principle, one should also retain c-quark events among those producing a detached vertex, eventually combining the corresponding rates with those for b-quarks, according to the values of efficiency and purity of the experimental analyses. In fact, the lifetime of c-quarks is finite (around 1/3 of that of the b's) and is thus responsible for secondary vertices. Therefore, one should expect that, for values of the gluino lifetime around $1/3\tau_b$ (and below), charmed hadrons can represent an additional background from ordinary QCD to the SUSY signal and the actual positions of the decay vertices of c- and b-quarks can partially overlap. This is the reason why the algorithms determining the efficiency/purity of flavour tagging used by the LEP I collaborations contain a multiplicity selection rule (the number of tracks produced being higher for b's than for c's) [44]. The values of purity obtained in a single b-tag at LEP I, around 95% or more [4,5,44], imply that above the b-selection cuts (in multiplicity and decay distance) the ordinary QCD contribution is indeed almost entirely due to decaying bottom hadrons, whereas below those cuts charmed hadrons are mainly responsible for secondary vertices. In other terms, the c- and b-contributions would enter in our analysis 'separately' from each other

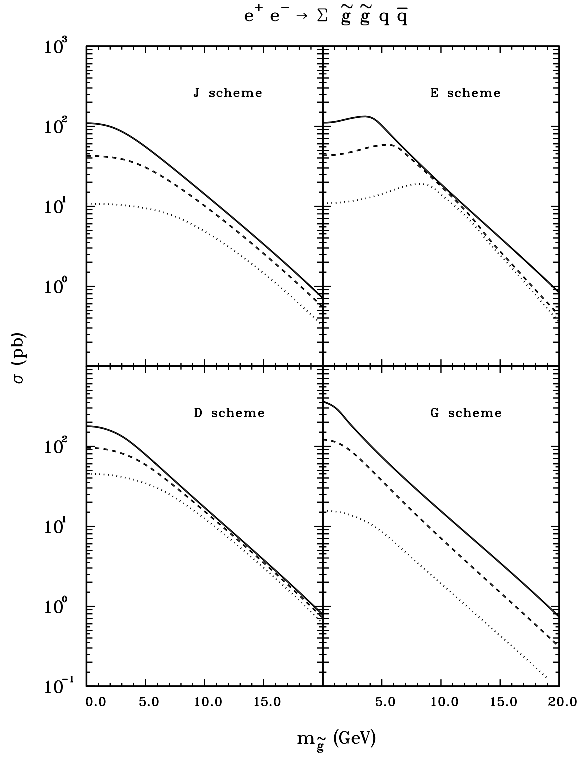

Fig. 4. Cross sections for $2q2\tilde{g}$ production. A sum over the quark flavours $q = u, d, s, c$ and b is implied. Curves are given as function of the gluino mass, for the four different algorithms J, E, D and G introduced in the text and three choices of the jet-scheme resolution parameter. The combinations are: $y^J_{\rm cut} = 0.01(0.02)[0.04]$, $y^E_{\rm cut} = 0.01(0.02)[0.04]$, $y^D_{\rm cut} = 0.002(0.004)[0.008]$, $y^G_{\rm cut} = 0.02(0.04)[0.08]$, in *continuous*(*dashed*)[*dotted*] *lines*

into the QCD background to SUSY signals, the relative small contamination being eventually established by the experimental tagging strategy. For reasons of space, in the following we will illustrate the interplay between gluinos and bottom quarks only. However, it must always be intended that in presence of a short decay distance and/or a low secondary vertex multiplicity the actual rates from ordinary QCD events will in the end need the inclusion of the mentioned corrections due to the differences between c- and b-quarks.

In Fig. 4 we study the effect of a non–zero gluino mass in the total cross section of the process $e^+e^- \to qq\tilde{g}\tilde{g}$ for the schemes described in Sect. 2 and for different values of the $y_{\rm cut}$ parameter. For $m_{\tilde{g}} \gtrsim 5$ GeV the cross section falls exponentially in the J and D schemes. For the G scheme the exponential behaviour starts somewhat earlier, at $m_{\tilde{g}} \gtrsim 2$ GeV, and for the E scheme later, when $m_{\tilde{g}} \gtrsim 7$ GeV. We already know that SUSY rates certainly compare rather poorly to both the $q\bar{q}gg$ and $q\bar{q}q'\bar{q}'$ contributions, if all quark flavours are retained and energy ordering is adopted [39]. Nonetheless, one of the salient features in Fig. 4 is that for $m_{\tilde{g}} \lesssim 10$ GeV the mass suppression on the SUSY rates is always less than one order of magnitude. This is true independently of jet algo-

[4] We assume the rate due to misidentification of gluons and massless quarks as heavy partons negligible [4]

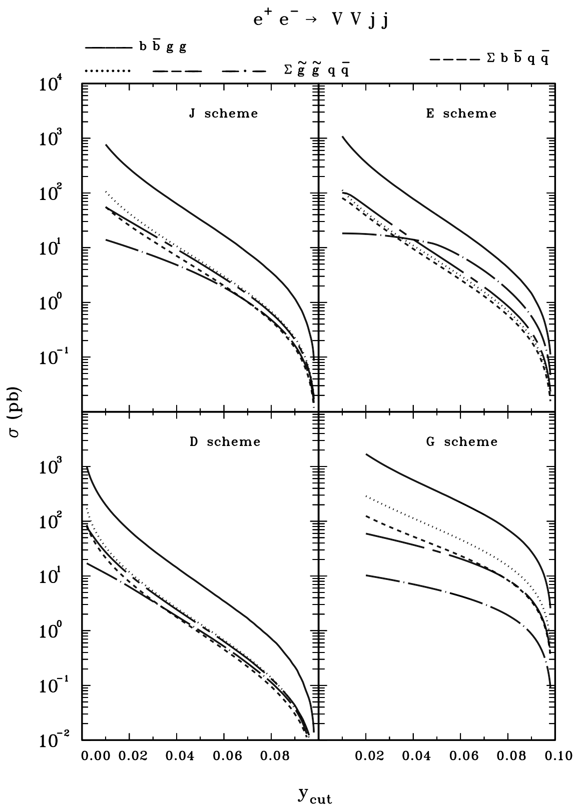

Fig. 5. Cross sections for the three different contributions to 4jet production in QCD+SUSY, in case of vertex tagging. Quark flavours are intended as summed over $q = u, d, s, c$ and b. Curves are given as function of the jet-scheme resolution parameter, for the four different algorithms J, E, D and G introduced in the text and three different values of gluino mass: $m_{\tilde{g}} = 1(5)[10]$ GeV, in *dotted*(*chain-dashed*)[*chain-dotted*] *lines*

rithm and for three typical values of the resolution parameter $y_{\rm cut}$. In any case the contribution for $m_{\tilde{g}} > 10$ GeV begins to be very small. Therefore, in the remainder of the paper we will confine ourselves to gluino masses up to 10 GeV only. At this point, one should recall that the ordinary QCD production rates are much larger than the SUSY ones displayed in Fig. 4. For example, when no vertex tagging is exploited and all flavours are retained in the sample, at the minimum of the $y_{\rm cut}$'s used there, one gets $\sigma(q\bar{q}gg) = 4153(4498)[5862]\{9312\}$ pb and $\sigma(q\bar{q}Q\bar{Q}) = 187(217)[300]\{548\}$ pb, in correspondence of the J(E)[D]{G} scheme. A similar pattern in the relative composition of 4jet events persists also at larger values of the resolution parameter.

The cross sections as a function of $y_{\rm cut}$ for the different subprocesses yielding two displaced vertices are presented in Fig. 5. The ratio between SUSY and pure QCD events is clearly improved, so that $b\bar{b}q\bar{q}$ and gluino rates now compare to each other. The largest contribution still comes from the subprocess $e^+e^- \to b\bar{b}gg$, which is almost one order of magnitude larger than the other two in the whole range of $y_{\rm cut}$. How to ameliorate this situation will be discussed below. The gluino rates are shown for three reference masses, $m_{\tilde{g}} = 1, 5, 10$ GeV. The pattern recognised in Fig. 4 as a function of the gluino mass is also visible in Fig. 5 for the $y_{\rm cut}$ dependence. That is, as the gluino mass increases the production rates get smaller, however still remaining within the same order of magnitude if $m_{\tilde{g}} \lesssim 10$ GeV. In practice, gluinos in the mass range up to 10 GeV have all sizeable production rates at LEP I in the jet schemes considered for usual values of the jet resolution parameters. This is indeed encouraging, as this means that the 4jet sample could well be sensible to values of $m_{\tilde{g}}$ larger than those usually considered (i.e., of the order of the b-mass or below).

Figure 6 shows the improvements that can be achieved with heavy flavour tagging combined with the typical kinematic behaviour of gluino events, see [3]. For reference, the gluino mass has been fixed at 5 GeV, though the main features of the plots do not depend on $m_{\tilde{g}}$ as these are connected only to the fact that gluinos are always secondary products. Only the D scheme is shown, for the other schemes exhibit very similar behaviour. The variable Y_{ij} is the invariant scaled mass

$$Y_{ij} = \frac{(p_i + p_j)^2}{s}, \qquad (9)$$

where s is the centre of mass energy squared ($s = M_Z^2$) and the indices ij label the jets as follows: (12) refer to the two vertex tagged jets, and to the most energetic jets in the case of energy ordering; (34) corresponds to the two remaining jets. The distributions are normalised to one. Note that in Fig. 6a the $2\tilde{g}2q$ and $2b2q$ events are peaked at low Y_{12} while the $2b2g$ events are evenly distributed. The peak in the first two cases is easily understood as it comes from the propagator $g^* \to b\bar{b}/\tilde{g}\tilde{g}$, which is not present in the third case (the tagged jets there come always from the Z decay). The long tail of the $2b2q$ spectra comes from the fact that there can be 'mis–tags' of b's coming from the Z propagator. The peak for $2q2\tilde{g}$ is even narrower, as the two gluinos are always produced through gluon splitting, apart from a small contamination of mis-tags coming from $2b2\tilde{g}$. The strategy is now clear: for $Y_{12} < 0.2$ most of the SUSY signal is retained while $2b2q(2b2g)$ events are reduced roughly by a factor of two(four). In contrast, when energy ordering is performed (Fig. 6b) this effect is washed out, as all the distributions have a similar shape and no useful cut can be devised. Note that the distributions are finite due to the masses of the tagged jets so that loop corrections will not change significantly the behaviour presented here. The situation is even better if we look at Fig. 6c: the distribution for $2q2\tilde{g}$ is flat and the other two distributions are peaked at $Y_{34} = 0$. This effect is just the complementary of Fig. 6a: the (34) jets come from the Z propagator in the $2q2\tilde{g}$ events while show the peak of the gluon splitting for the other two cases. Again, when energy ordering is performed, Fig. 6d, the effect is wiped off. Therefore, we adopt the following requirements to optimise the SUSY signal over the ordinary QCD background: $Y_{12} < 0.2$ and $Y_{34} > 0.1$. Note that the use of the tagging procedure has been crucial for such an achievement. These simple invariant mass distributions serve the

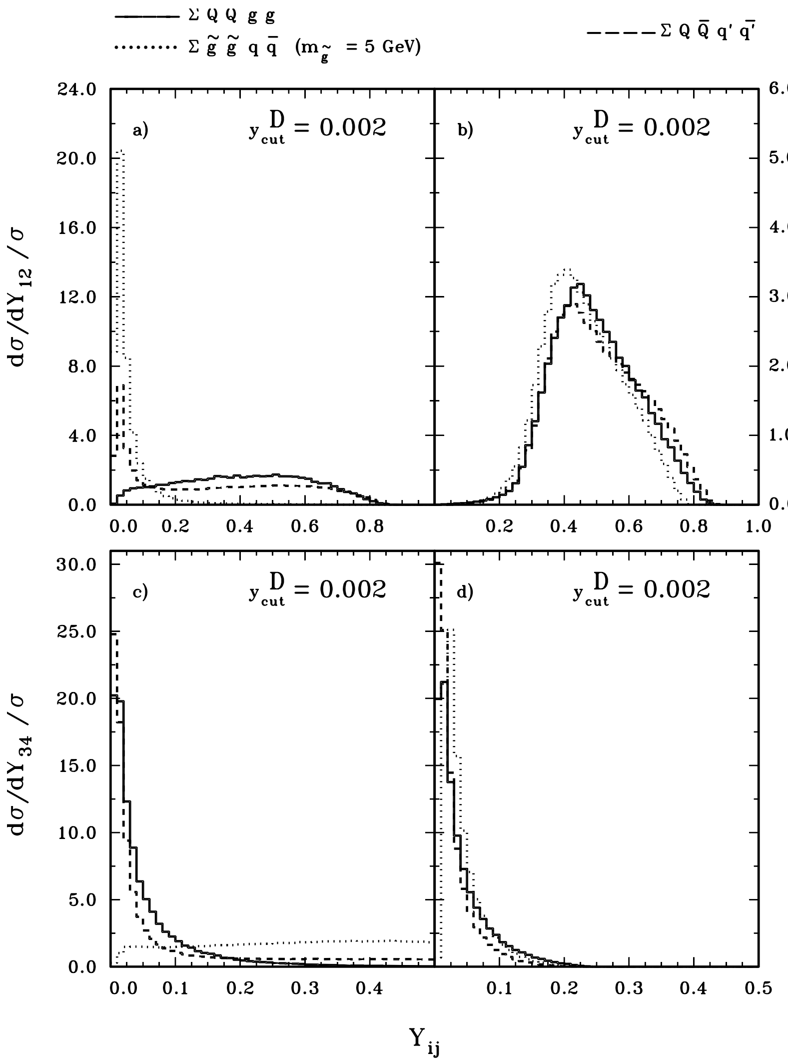

Fig. 6. **a** Differential distributions in the rescaled invariant mass of the vertex tagged pair of jets for the three different contributions to 4jet production in QCD+SUSY. Quark flavours are intended as summed over $q = u, d, s, c$ and b. Curves are given for the algorithm D introduced in the text, for the minimum of the jet-scheme resolution parameter $y_{\rm cut} = 0.002$. The gluino mass is set equal to 5 GeV. Distributions are normalised to unity. **b** The same distributions in case of energy ordering of the jets. Here, the invariant mass corresponds to that of the two most energetic jets. An additional sum over $q' = u, d, s, c$ and b is implied. **c** Same differential distributions as in **a** in the rescaled invariant mass of the 'untagged' pair of jets. **d** Same distributions as in **b** in the rescaled invariant mass of the 'untagged' pair of jets

purpose of reducing the ordinary QCD rates in case of $\tau_{\tilde{g}} \lesssim \tau_b$, as it can happen when $m_{\tilde{g}} \lesssim 1.5$ GeV. For $m_{\tilde{g}} \gtrsim 3.5$ GeV, where $\tau_{\tilde{g}} \gg \tau_b$ and the gluino and quark vertices are in principle well distinguishable, the kinematic distributions would clearly help to elucidate the underlying SUSY dynamics.

In Fig. 7 we show the different contributions to the total cross section in our tagging procedure like in Fig. 5, but with the improved sample. The $2b2g$ event rates, which were one order of magnitude larger than those of the other two partonic components, have been greatly reduced. All contributions are now comparable (at least for $m_{\tilde{g}} = 1$–5 GeV). For $m_{\tilde{g}} \gtrsim 5$ GeV the ordinary QCD events can be most likely eliminated from the sample already on a displaced vertex basis, by asking, e.g., that the decay length is much longer than 0.3 mm. However, for completeness we report the rates for large gluino masses too, as the tagging procedure could be complicated by the fact that a large part of the vertex tagged hadronic sample at LEP I has been collected via a bi-dimensional tagging [45]. Therefore, projections of different decay lengths d could well appear the same on the reproduced event plane. It is also worth recalling that the fact that gluinos are electrically neutral whereas quarks are charged can hardly be useful in 4jet analyses as there is extremely low efficiency in measuring the total jet charge, especially in multijet events. That explains why, for instance, this difference is not used to discriminate partonic compositions in ordinary 4jet events (as gluons too are neutral). In summary, we have shown that it is feasible to significantly enhance the signal of possible light gluino species over the QCD background using tagged samples with the help of elementary kinematical distributions.

5.2 Missing energy distributions

In this section we study the decays rates of SUSY events, for three representative values of the gluino mass which yield sizeable production rates. In particular, we will investigate the spectrum in missing energy inside the gluino jets, trying to establish the quantitative relevance in the

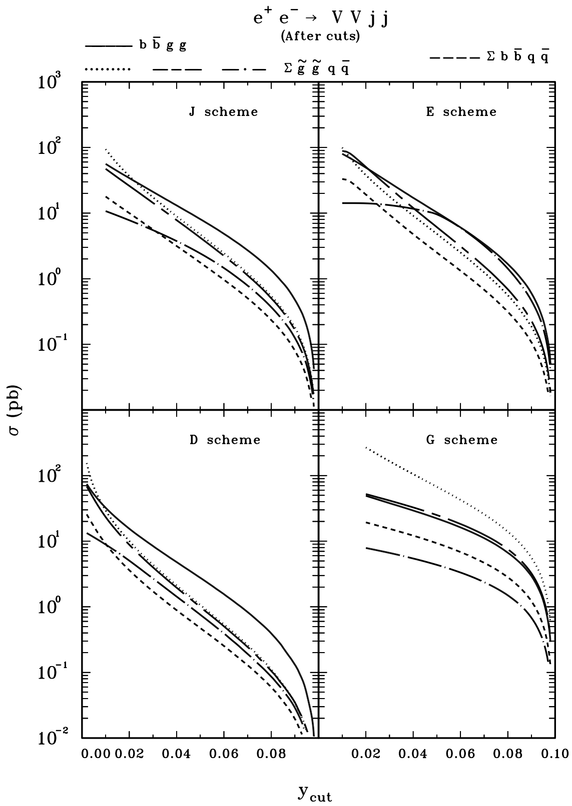

Fig. 7. Cross sections for the three different contributions to 4jet production in QCD+SUSY, in case of vertex tagging, after the kinematical cuts. Quark flavours are intended as summed over $q = u, d, s, c$ and b. Curves are given as function of the jet-scheme resolution parameter, for the four different algorithms J, E, D and G introduced in the text and three different values of gluino mass: $m_{\tilde{g}} = 1(5)[10]$ GeV, in *dotted*(*chain-dashed*)[*chain-dotted*] *lines*

total SUSY sample of hadronic events carrying an energetic imbalance that does not meet the usual trigger thresholds of the LEP I detectors.

In discussing the possible decay modes of the gluino, two assumptions need to be made. The first is the condition of R-parity conservation. The second is the choice of the lowest mass Supersymmetric particle. R-parity, defined to be even for ordinary particles and odd for their Supersymmetric counterparts, needs to be preserved if lepton and baryon numbers are exactly conserved. This implies that the lightest Supersymmetric particle is exactly stable. In this paper we shall take it for granted that the neutralino (photino) is the lowest mass Supersymmetric particle. Failing this condition, the next likely choice would be the case where the scalar neutrinos are lower in mass. However, very light doublet sneutrinos are excluded by the Z width constraints [15].

The choice of the scalar quark masses $\tilde{M}_L$ and $\tilde{M}_R$ [1] affects the gluino branching ratios. In particular, considering only one flavour of massless quarks and assuming that the photino is massless, the ratio between the widths of the two dominant gluino decay modes is given by:

$$\frac{\Gamma(\tilde{g} \to g\tilde{\gamma})}{\Gamma(\tilde{g} \to q\bar{q}\tilde{\gamma})} = \frac{3\alpha_s}{4\pi}\frac{(\tilde{M}_R^2 - \tilde{M}_L^2)^2}{(\tilde{M}_L^4 + \tilde{M}_R^4)}. \qquad (10)$$

Therefore, the quark-antiquark-neutralino decay channel is dominant over the gluon-neutralino one. However, in some SUSY models the L and R mass eigenstates may differ by a factor of two or even more, such that $(\tilde{M}_R^2 - \tilde{M}_L^2)^2 /(\tilde{M}_L^4 + \tilde{M}_R^4) \gtrsim 1/2$ [46]. Furthermore, as the photino mass approaches that of the gluino, $m_{\tilde{\gamma}}/m_{\tilde{g}} \to 1$, the three-body decay mode suffers a further suppression, which goes as $(1 - m_{\tilde{\gamma}}/m_{\tilde{g}})^2$. A more extensive review of the gluino decay channels can be found, for example, in Sect. 3.4 of Haber and Kane [46] (see also references therein).

We have computed the relevant decay currents by using helicity amplitude techniques, and incorporated these into a complete matrix element for gluino production and decay, over the appropriate phase space. In doing so, two different formalisms were employed: the usual helicity projector method [47] and the techniques of [48]. The results obtained with the two methods agree for any polarisation state if in the latter formalism one modifies the helicity projections to coincide with the physical choice along the direction of the final partons.

Before studying the decay spectra, a few comments are in order concerning the fragmentation of a gluino. As already mentioned, a gluino would appear at the end of a hadronisation process confined into a bound state. Now, the decay kinematics of R-hadrons is in principle different from that of free $\tilde{g}$'s. However, if the gluino is sufficiently heavy (say, $m_{\tilde{g}} \gtrsim 3.5$ GeV [46]), the phenomenology of the decay products of such R-hadrons would be similar to that of unbounded gluinos. In particular, the basic result is that the $\tilde{\gamma}$ energy spectrum roughly agrees with that produced by a freely decaying $\tilde{g}$ as long as $m_{\tilde{\gamma}}/m_{\tilde{g}}$ is not too close to one [49]. For lighter gluinos the analysis is less straightforward. However, according to [50], it is reasonable to expect that these SUSY hadrons would again decay similarly to free gluinos, provided that $m_{\tilde{g}}$ is replaced by an 'effective' R-hadron mass equal to $\approx 0.75\ m_{\tilde{g}}$. For our purposes, we assume that the mass appearing in the decay spectra is in fact that of the SUSY parton in the mass range (ii), whereas in the interval (i) it represents the mentioned effective mass. Furthermore, on the one hand, we confine ourselves to values of $m_{\tilde{\gamma}}$ strictly smaller than $m_{\tilde{g}}$ in order to maintain valid our approximation over the range $m_{\tilde{g}} \lesssim 1.5$ GeV; on the other hand, we will push the ratio $m_{\tilde{\gamma}}/m_{\tilde{g}}$ up to 3/4 in order to maximise the amount of missing energy carried away by the undetected photino.

The results we have obtained for the energy distribution of the missing energy after the two decays are displayed in Figs. 8a–c and Figs. 9a–c (in correspondence of the two possible decays). The crucial point is that the amount of missing energies produced could be so large that SUSY events of the type $q\bar{q}\tilde{g}\tilde{g}$ are not recognised as 4jet events. In fact, experimental analyses have a minimal hadronic energy cut on each of the four jets, in order to reduce the background due to poorly reconstructed events.

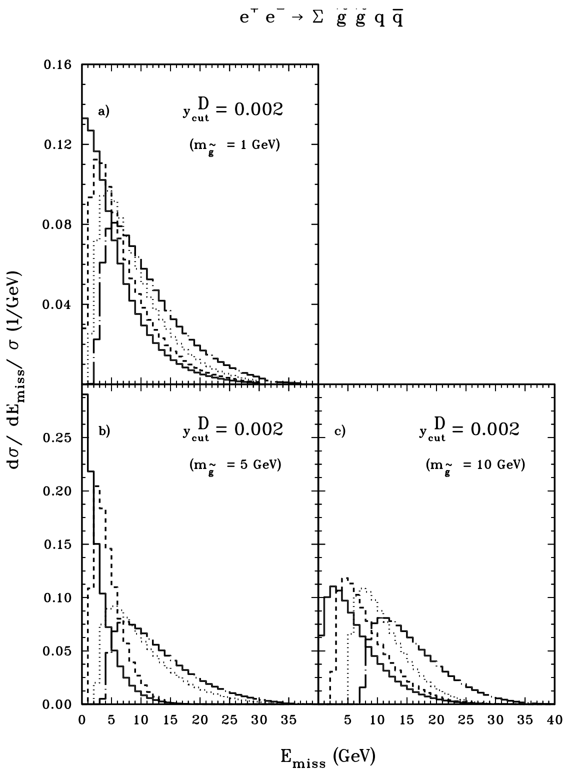

Fig. 8. Differential distributions in the missing energy of the 'gluino jet' after the SUSY decays $\tilde{g} \to q\bar{q}\tilde{\gamma}$, where $\tilde{\gamma}$ represents the 'photino'. The masses of this latter are $m_{\tilde{\gamma}} = 0$ (*continuous*), $1/4m_{\tilde{g}}$ (*dashed*), $1/2m_{\tilde{g}}$ (*dotted*) and $3/4m_{\tilde{g}}$ (*chain-dotted*). The gluino masses are: $m_{\tilde{g}} = 1$ **a**, 5 **b** and 10 **c** GeV. The mass refers to the 'effective' mass of the bound gluino if $m_{\tilde{g}} = 1$ GeV. No kinematical cut is here applied

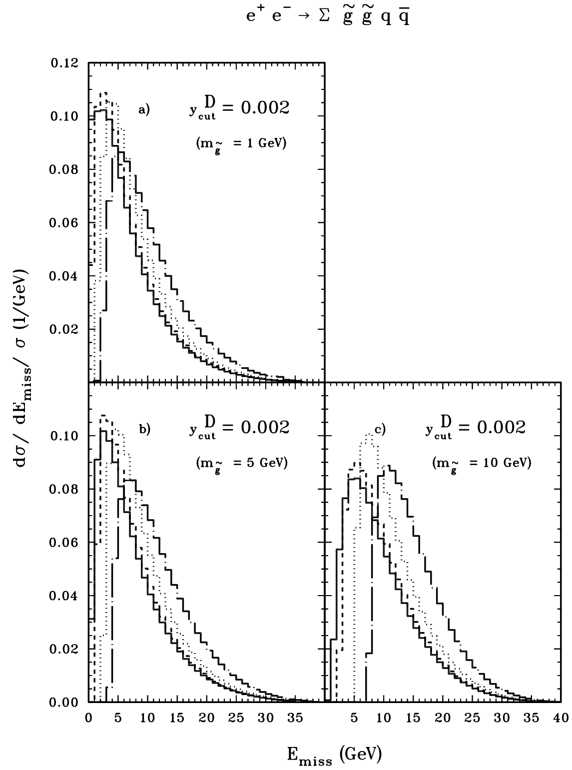

Fig. 9. Differential distributions in the missing energy of the 'gluino jet' after the SUSY decays $\tilde{g} \to g\tilde{\gamma}$, where $\tilde{\gamma}$ represents the 'photino'. The masses of this latter are $m_{\tilde{\gamma}} = 0$ (*continuous*), $1/4m_{\tilde{g}}$ (*dashed*), $1/2m_{\tilde{g}}$ (*dotted*) and $3/4m_{\tilde{g}}$ (*chain-dotted*). The gluino masses are: $m_{\tilde{g}} = 1$ **a**, 5 **b** and 10 **c** GeV. The mass refers to the 'effective' mass of the bound gluino if $m_{\tilde{g}} = 1$ GeV. Normalisations are to unity. No kinematical cut is here applied

The E_{miss} spectra are shown for four kinematical decay configurations: a massless photino, and a massive one with $m_{\tilde{\gamma}} = n/4m_{\tilde{g}}$, with $n = 1, 2, 3$, and for three gluino masses $m_{\tilde{g}} = 1, 5, 10$ GeV. It is clear from both Fig. 8a–c and 9a–c that the missing energy spectrum gets harder as $m_{\tilde{g}}$ and $m_{\tilde{\gamma}}$ increase in both decay channels considered. The effect is common to all algorithms. For $m_{\tilde{g}} = 1$ GeV the mean value of the missing energy is always below 10 GeV in both decay channels, and it can grow up to more than 15 GeV if $m_{\tilde{g}} = 10$ GeV and $m_{\tilde{\gamma}} = 7.5$ GeV. Under such conditions, it could be argued that $q\bar{q}\tilde{g}\tilde{g}$ events can pass unobserved as actual 4jet events if tight constraints are implemented on the missing mass energy of the hadronic event sample.

6 Summary and conclusions

In this paper we have studied the production and decay rates of $e^+e^- \to q\bar{q}\tilde{g}\tilde{g}$ events at LEP I, where $\tilde{g}$ represents a relatively light (up to 10 GeV in mass) and long-lived (up to 10^{-9} sec in lifetime) gluino, and compared these to the yield of ordinary QCD events of the type $e^+e^- \to q\bar{q}gg$ and $e^+e^- \to q\bar{q}q'\bar{q}'$, involving quarks q and gluons g. The presence of such SUSY events in 4jet samples at LEP I has been advocated in the past years to explain the disagreement between the values of the strong coupling constant α_s as measured from the deep-inelastic scattering and the Z-peak data. This was further motivated by the initial discrepancy between the QCD predictions for the colour factors C_A, C_F and T_F and their actual measurements obtained in earlier analyses [25] by the LEP collaborations, as these constants are sensitive to additional SUSY contributions. The claim about the possible existence of gluinos in LEP I data has apparently become less convincing during the recent two or three years, as the experimental and theoretical analyses of the data have reached a higher level of sophistication and precision. Very recent studies seem to exclude gluinos with masses up to 6.3 GeV. Although such results represent clear progress towards settling the ongoing dispute about the existence of SUSY signals at LEP I, we have outlined here a complementary approach guided by two considerations.

First, in all the mentioned analyses no vertex tagging was exploited in assigning the momenta of the jets to the corresponding partons from which the former originate. The study of 4jet events showing two secondary vertices produced in the decay of c- and b-quarks has in fact been proved to be successful in reducing the error on the QCD colour factor which is most sensitive to the possible presence of light gluinos (that is, $T_R = N_F T_F$). Furthermore, we have also shown that simple kinematic distributions (such as the invariant masses of the two vertex tagged jets and of the remaining two) can effectively help to enrich significantly the 4jet samples of gluino events (if existing). In fact, the latter, on the one hand, should yield displaced vertices and, on the other hand, are always produced as secondary partons (contrary to heavy quarks).

Second, the validity of the result quoted by the ALEPH collaboration (the most constraining one) could be undermined by the fact that, in their procedure of selecting candidate 4jet samples, events carrying a large fraction of missing energy were not included. As a matter of fact, gluinos (or better, R-hadrons, in which the SUSY partner of the gluon is confined) should predominantly decay into 'photinos', which escape detection. Indeed, there are kinematic configurations in which the ratio between the gluino and photino mass is such that the missing energy is rather large and, conversely, the left-over one for the hadronic system arising from the SUSY decay is rather small, such that these events might not pass the experimental 4jet resolution and selection criteria.

In the above context, we believe to have obtained interesting results for future studies. In fact, we have shown that, in the vertex tagged sample of 4jets, SUSY events become comparable to the rates of ordinary QCD events for gluino masses up to about 10 GeV, thus well beyond the bounds presently set on this quantity. Furthermore, we have indicated that the latter cannot be reliable if the photino mass is not negligible compared to that of the gluino. Therefore, we conclude that experimental analyses based on our approach should help in clarifying the present debate, either contradicting the present bounds on the gluino mass or improving these by extending the experimental coverage of the so-called 'light gluino window'. For example, over the mass region $m_b < m_{\tilde{g}} \lesssim 10$ GeV, SUSY rates should still be sizable and yield an unmistakable signature with two hard jets, two soft ones, large missing energy, two detached vertices with $c\tau \gtrsim 0.3$ mm, a neat peak in the invariant mass of the vertex tagged dijet system and a very flat distribution in the mass of the other two jets.

In carrying out our study we have resorted to parton level calculations, which include all masses of primary and secondary partons exactly. Although we have not implemented a full Monte Carlo procedure including the fragmentation of the gluinos into hadrons or the decay of the latter into jets and missing particles, we have used analytic approximations which should mimic well the actual gluino phenomenology to a degree of accuracy compatible with that of the current experimental analyses. In this respect, we have indicated possible signatures of gluinos decaying inside the LEP detectors, as a function of both the mass and the lifetime of the SUSY particle.

Before closing, we would like to point out a few crucial aspects of our work. First, contrary to many previous studies (which did not exploit vertex tagging and/or kinematical cuts) in which the gluino component represents an effect of just a few percent (thereby being of the same order as next-to-leading and/or hadronisation corrections), we have been concerned with SUSY rates that are always comparable or even larger than those produced by pure QCD events. Therefore, the inclusion of the mentioned corrections will not spoil our results. Second, for values of gluino masses up to 10 GeV or so, our conclusions are essentially the same independently of the jet algorithm and of the value used for $y_{\rm cut}$ (although the actual cross sections and the behaviour of the distributions do certainly depend on them). In the end, the magnitude of higher order and hadronisation effects as well as experimental considerations will determine which algorithm and which resolution parameter are the most suitable to use, though the Geneva algorithm seems to be slightly favoured due to its special sensitivity to the actual number of active flavours and a smaller scale dependence in NLO corrections. Third, by adopting the current LEP I values of vertex tagging efficiency and luminosity, we should expect a statistically significant analysis, based on several thousands of doubly tagged 4jet events. Fourth, since those presented here are theoretical results from parton level calculations, they will necessarily have to be folded with detailed experimental simulations (including both fragmentation/hadronization and detector effects), such that one could even improve at that stage our procedure: for example, by exploiting various differences (in charge, mass, lifetime) that occur between heavy quarks and gluinos.

We finally remark that in the long term our arguments could well be of interest also to the SLC experiment at SLAC, as microvertex devices are installed there and they are known to have achieved by now a considerable tagging efficiency, so to hopefully compensate for the present lack of statistics of their data with respect to the LEP ones.

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